The Relationship Between $Q(x)$ and MATLAB’s erfc.

Recall that we have defined the function $Q(x)$ as the complementary cumulative distribution function for the standard normal distribution. That is,

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{1}{2}t^2\right) dt.$$ 

Also recall that MATLAB’s definition for the complementary error function erfc$(x)$ is

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-t^2) d\tau.$$ 

Via a simple change of variables ($t = \sqrt{2}\tau$) one can show that

$$Q(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

and that

$$Q^{-1}(x) = \sqrt{2} \text{erfc}^{-1}(2x).$$