Name: ________________________________

This is a open-book, open-notes exam. The use of electronic calculators is permitted. The exam lasts 75 minutes.

There are three questions on the exam. Do all your work on these pages and indicate your final answer clearly. For each of the problems I’ve provided an extra worksheet. There are also two extra pages at the back that you may use, and if necessary you may also use the backs of the sheets, but in either of these cases please mark clearly the problem with which that work is associated. Neatness and clarity are important and can influence your grade!

Each problem is weighted toward the final total as shown below.

Grades

1. __________________ (35 pts.)
2. __________________ (30 pts.)
3. __________________ (35 pts.)
Total ________________ (100 pts.)
1. **[30 points]** Suppose you make a set of measurements \( x(n), n = 0, \ldots, N - 1, \) where

\[
x(n) = a \cos \left( 2\pi \frac{k}{N} n \right) + b \sin \left( 2\pi \frac{k}{N} n \right) + w(n)
\]

with \( w(n) \) white Gaussian noise and with \( k \) an integer between 1 and \( N - 1 \).

(a) Find an MVUE for \( \theta = [a \ b]^T \).

Note that

\[
\sum_{n=0}^{N-1} \cos \left( \frac{2\pi kn}{N} \right) \cos \left( \frac{2\pi kn}{N} + \phi \right) = \frac{N}{2} \cos(\phi).
\]

(b) Based on your answer for part (a), suggest an estimator for

\[
P = \frac{a^2 + b^2}{2},
\]

which is the power of the signal portion of \( x(n) \). As \( N \to \infty \), what is the variance of this estimate?
EXTRA WORKSHEET for problem 1
2. [30 points] You measure $x(n) = A + w(n)$ for $n = 0$ and $n = 1$, where $A$ is an unknown constant and where $w(0)$ and $w(1)$ are independent, zero mean, Gaussian random variables with variances

$$VAR(w(0)) = 1$$
$$VAR(w(1)) = \begin{cases} 1, & 0 \leq A \geq 0 \\ 2, & A < 0 \end{cases}$$

(a) Find the Cramer Rao Lower Bound for unbiased estimators of $A$ based on $x(0)$ and $x(1)$.

(b) Show that an MVUE does not exist in this case.
EXTRA WORKSHEET for problem 2
3. [35 points] You have $N$ IID observations $\{x(0), \ldots, x(N-1)\}$ where

$$p(x(n); A, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left[-\frac{1}{2\sigma^2} (x(n) - A)^2\right].$$

Both the mean $A$ and the variance $\sigma^2$ are unknown. Find the Maximum Likelihood Estimate for the signal to noise ratio $\alpha = A^2/\sigma^2$. 
EXTRA WORKSHEET for problem 3
EXTRA WORKSHEET (indicate problem number clearly)
EXTRA WORKSHEET (indicate problem number clearly)