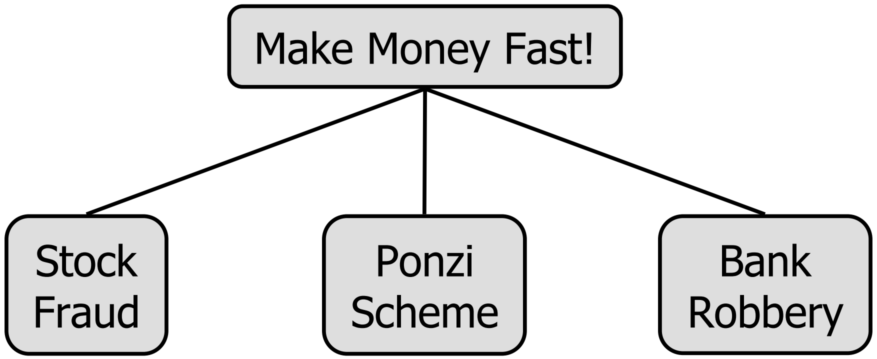
Trees and Codes



Binary Trees and Prefix Codes

- Imagine that you have an alphabet of symbols
  - e.g. A, B, …, Z, a, b, …, z, ‘ ’, ‘.’
- We wish to represent a string of these symbols as a string of bits
  - e.g. “This is a string of characters” becomes “01100111000101010111001”

Method 1: Use a fixed number for each symbol

- Map A -> 1, B -> 2, …
- "This is a string of characters" becomes a string of numbers
  - "20,34,35,…"
- I required commas to separate the characters!
  - Use a fixed width, padded with leading 0's instead
  - "020034035…"

Method 1: Fixed width

- How wide to my characters need to be
  - Using a string of bits
  - How many bits are needed to represent the largest character?
- ceil($\log_2$(n)) bits
- Use that many bits for each character
- This is the system used within the computer with 8 bits for each ASCII code

Method II: Prefix codes

- For each symbol, we’ll use a code with a special property
  - No code is the prefix of any other code
- How does this work?
- Decoding:
  - We read in the codes one bit at a time
  - When we have a code we recognise, it must be the end of a symbol
    - It cannot be part of a longer symbol because no code is the prefix of another code

Method II: Prefix codes

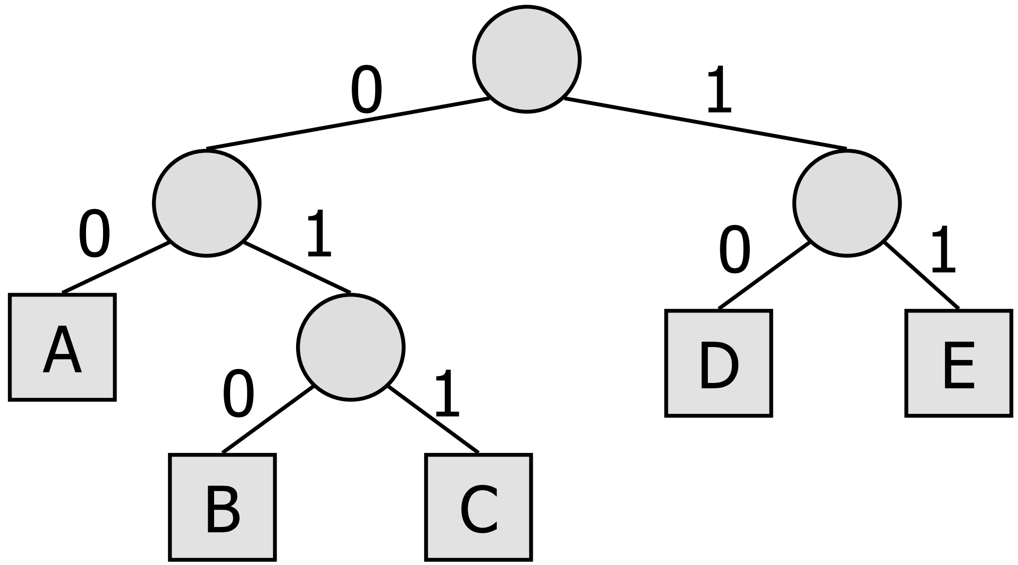
- Example
  - A:00, B:010, C:011, D:10, E:11
- ADBECABADE
- 0010010110110001000101 1
- A D B E C A B A D E
- The string decodes

Making a Prefix Code

- We want the code to be efficient
  - No strings longer than necessary
  - No wasted strings
- A code is a set of strings of binary digits, such that no string corresponding to one symbol is the prefix of a string corresponding to another symbol
- In a tree, leaf nodes have no children
  - No path from the root to a leaf is the prefix of a path from the root to another node

Binary Trees and Prefix Codes

- Binary trees are in one to one correspondence with Prefix Codes
  - A:00, B:010, C:011, D:10, E:11



Prefix Trees

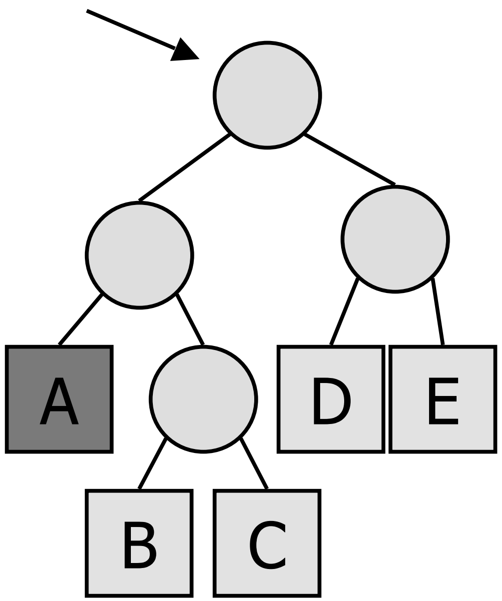
- Binary Trees
  - The left child corresponds to 0, the right to 1
- Each leaf contains a symbol
- The code for a symbol corresponds to the path from the root to the leaf containing that symbol

Encoding and Decoding

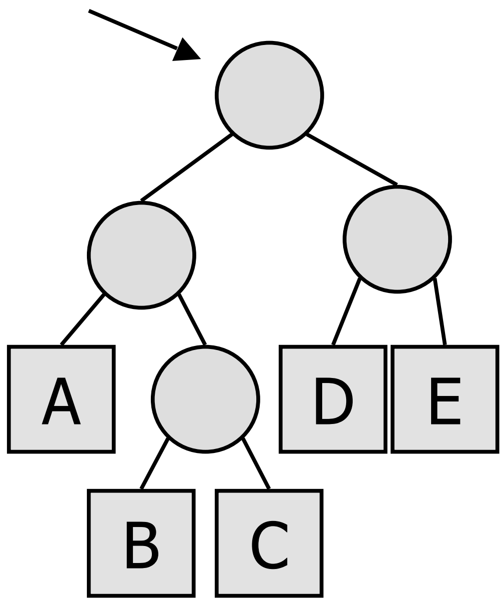
- Imagine the encoder and decoder running in parallel
- Encoding
  - Start from the root
  - While you are not at the symbol's leaf
    - If the symbol you wish to send is a left decendant, send 0 and move to your left child, else send 1 and move to your right child
- Decoding
  - Start from the root
  - While you are not at a leaf
    - Read a bit. If it is 0 then move to your left chile, else move to your right child

Encoding and Decoding: ACD

- Encoding:ACD

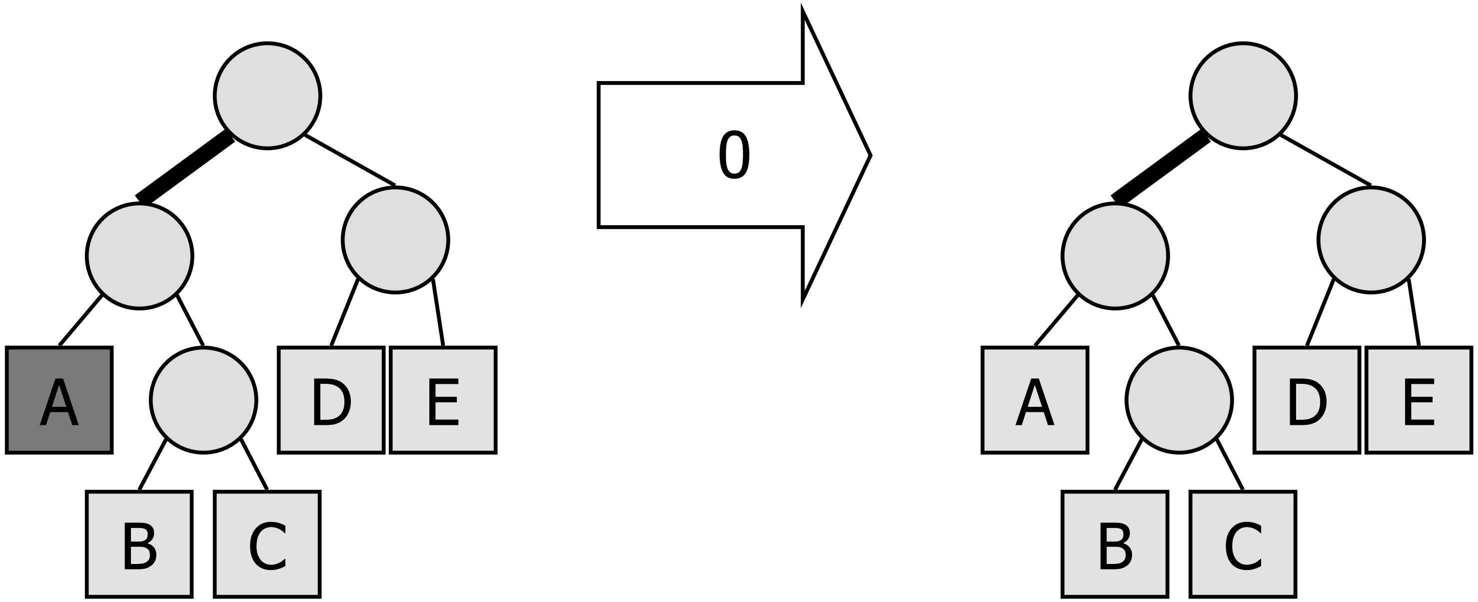


- Decoding:



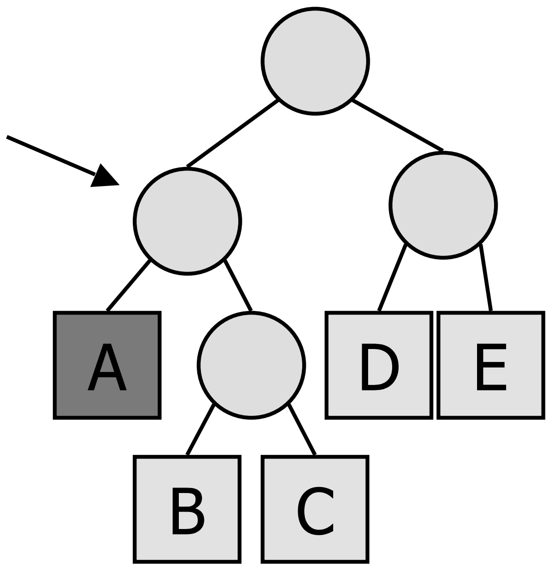
Encoding and Decoding: ACD

- Encoding:ACD
- Decoding:

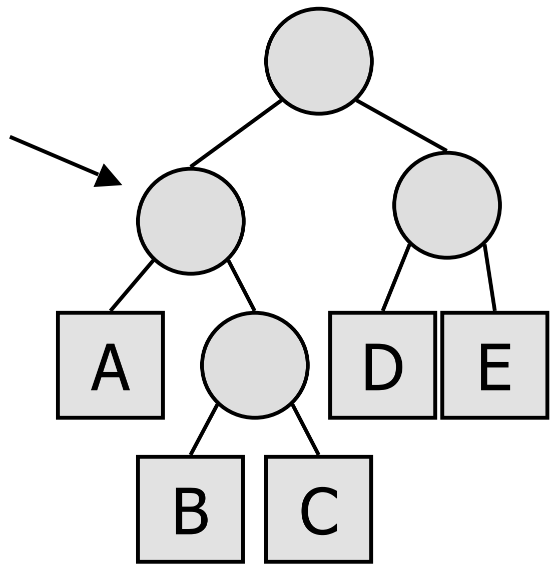


Encoding and Decoding: ACD

- Encoding:ACD

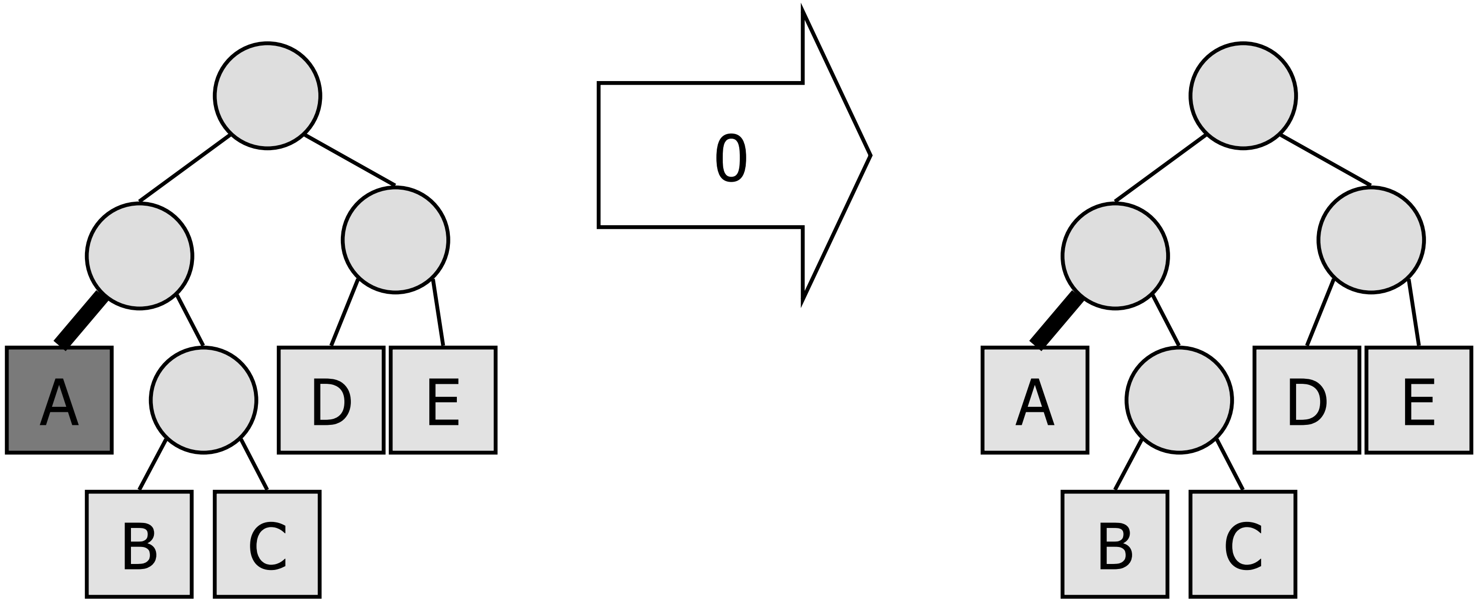


- Decoding:



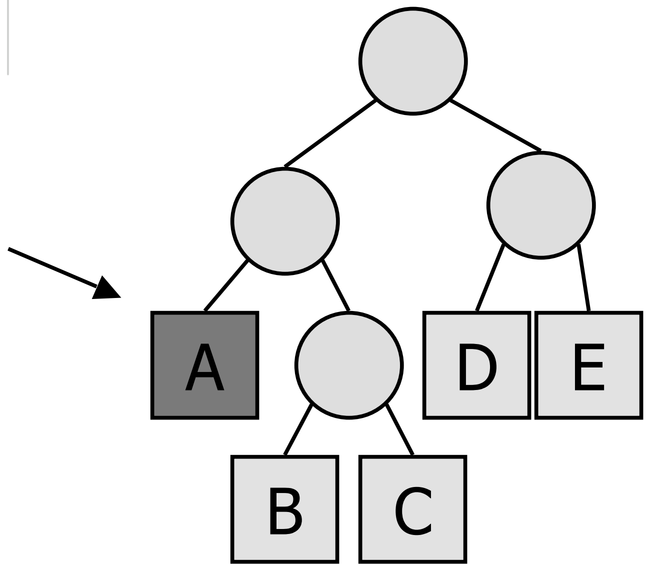
Encoding and Decoding: ACD

- Encoding:ACD
- Decoding:

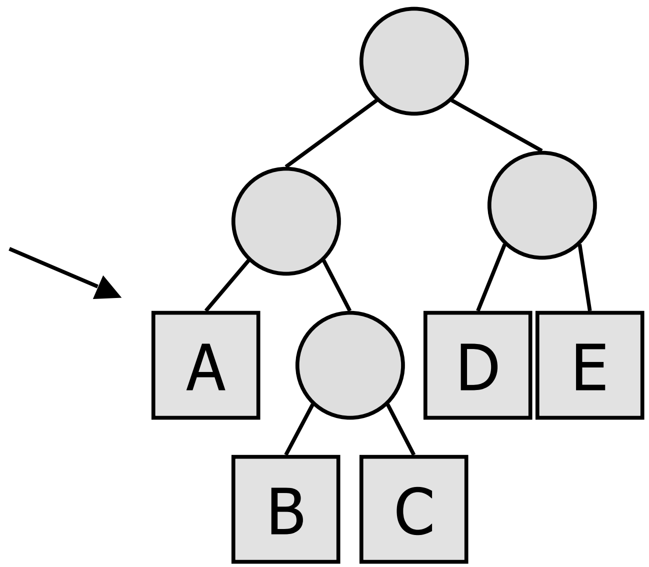


Encoding and Decoding: ACD

- Encoding:CD

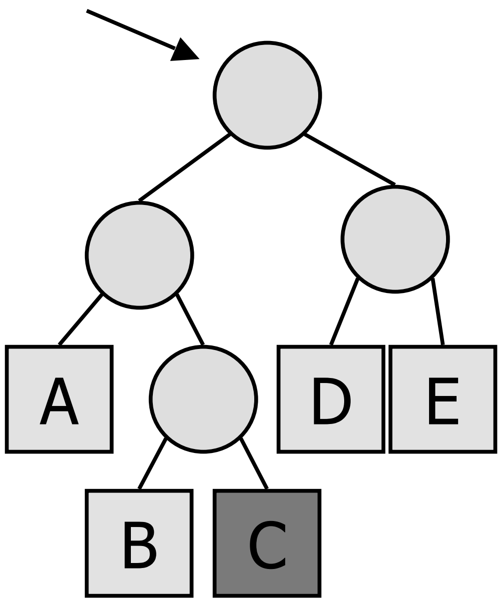


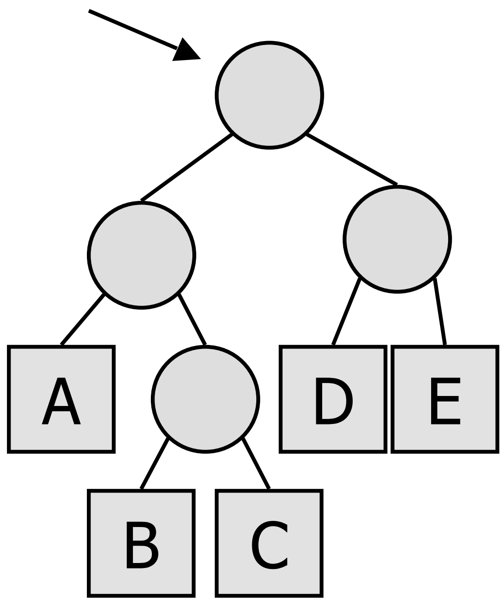
- Decoding:A



Encoding and Decoding: ACD

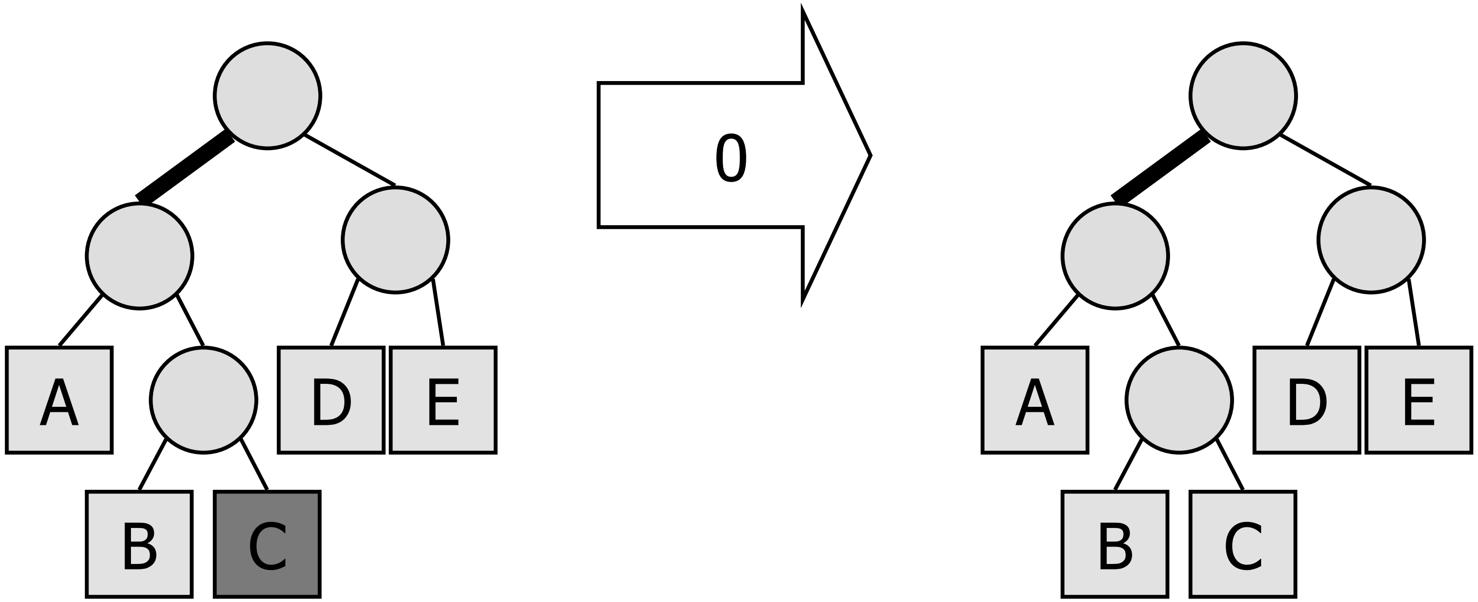
- Encoding:CD
- Decoding:A





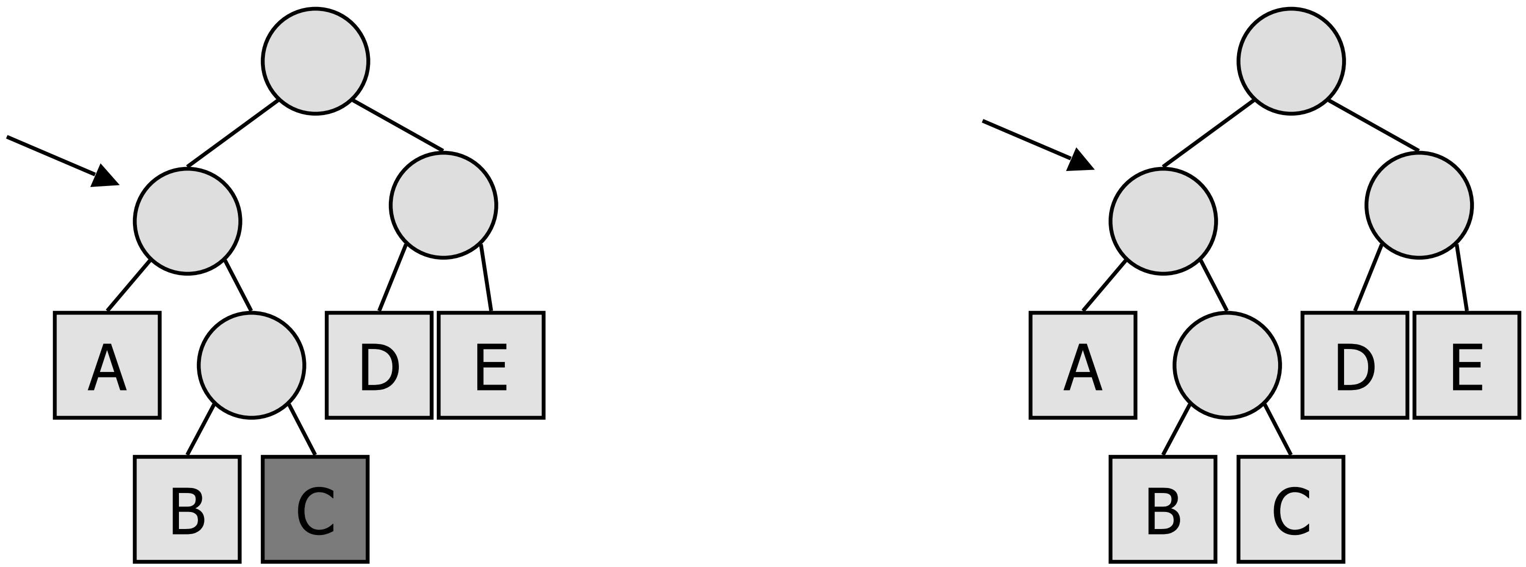
Encoding and Decoding: ACD

- Encoding:CD
- Decoding:A



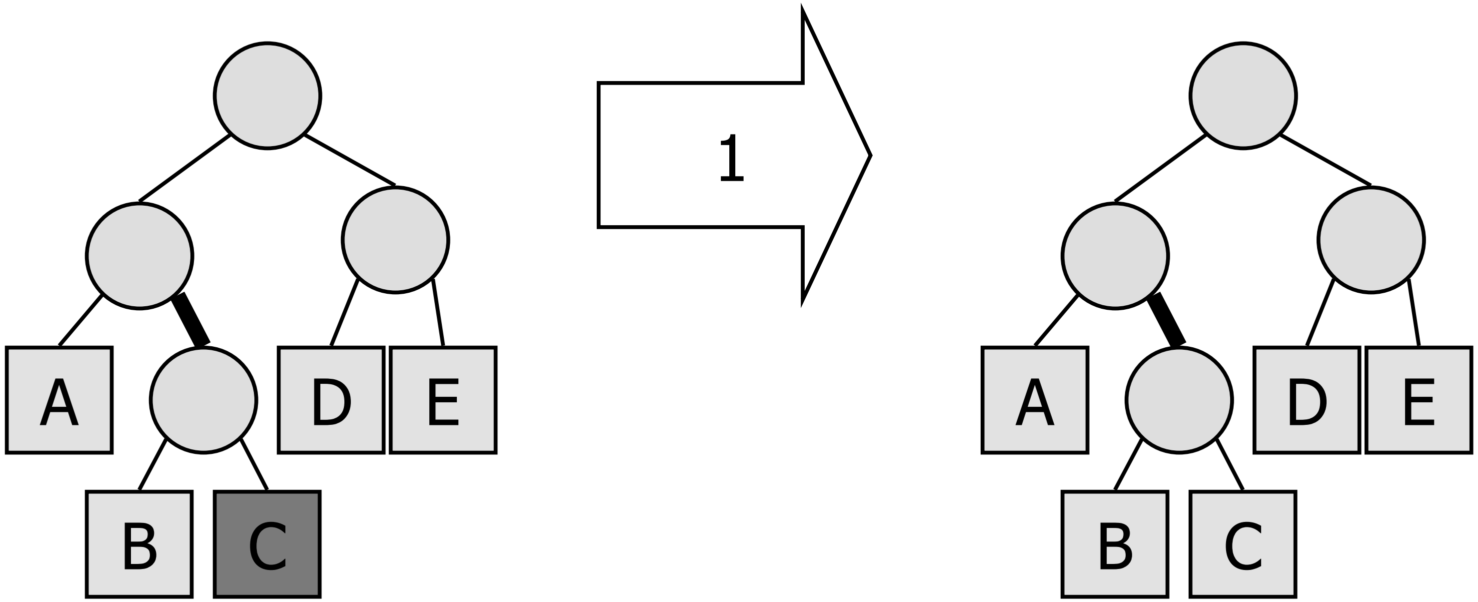
Encoding and Decoding: ACD

- Encoding:CD
- Decoding:A



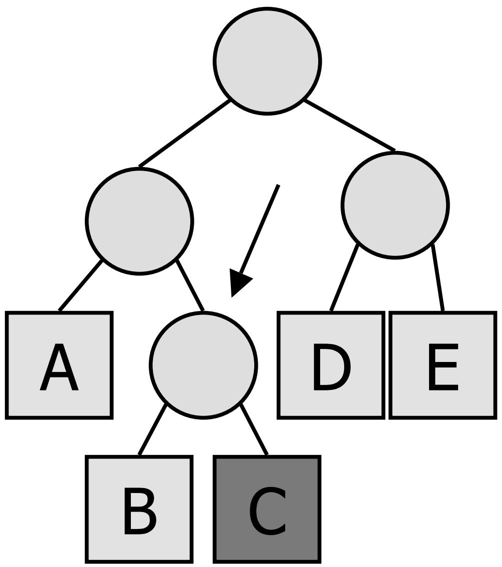
Encoding and Decoding: ACD

- Encoding:CD
- Decoding:A

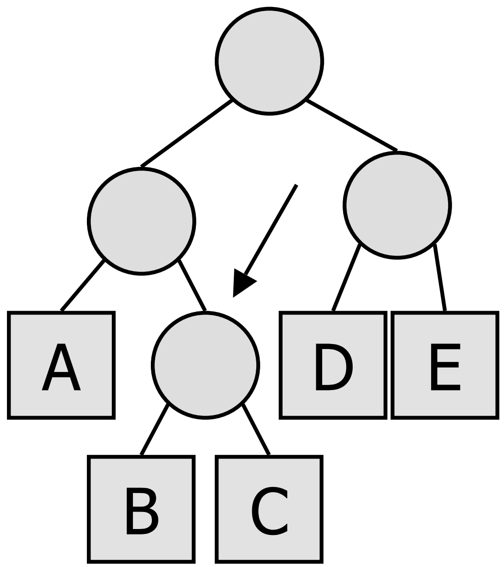


Encoding and Decoding: ACD

- Encoding:CD

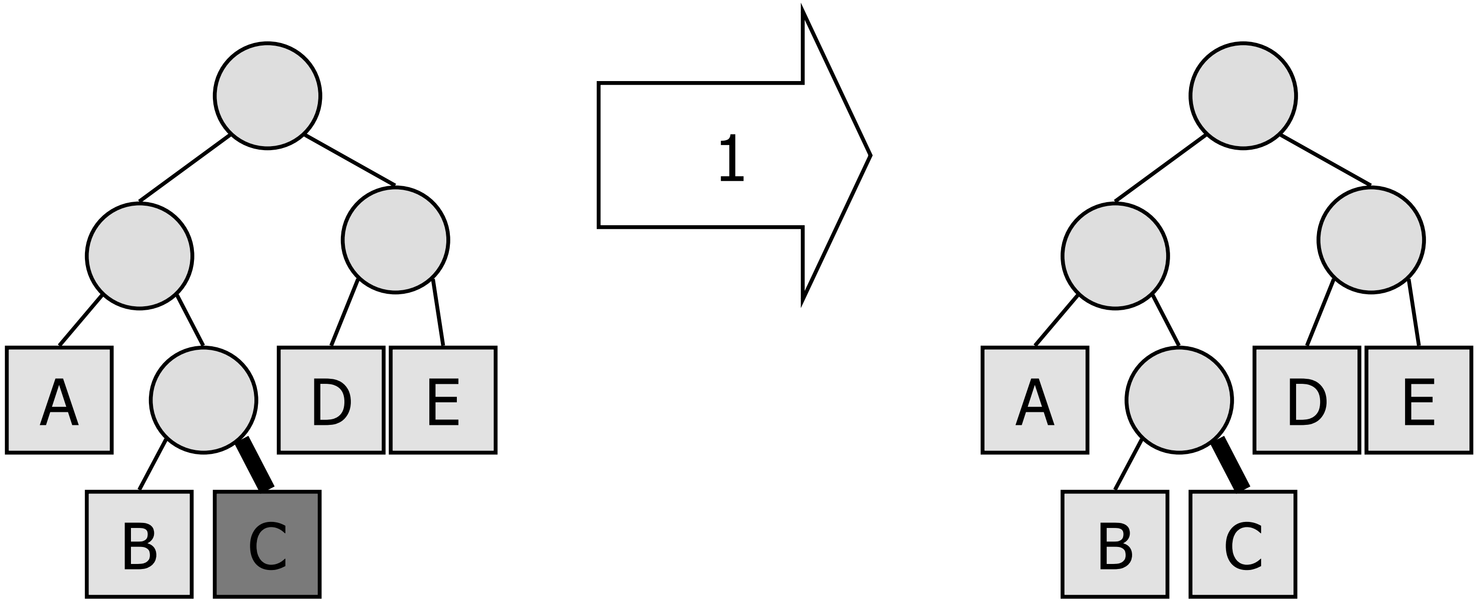


- Decoding:A



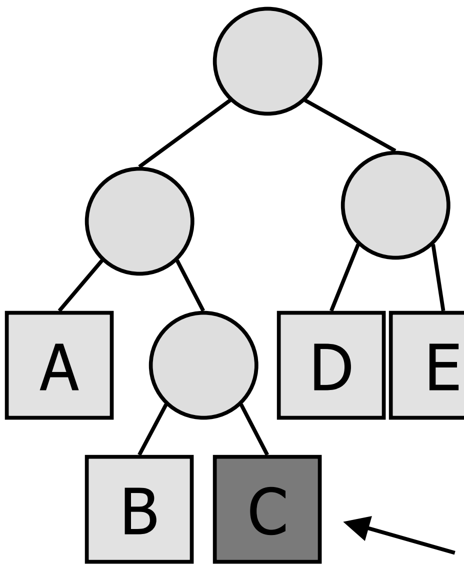
Encoding and Decoding: ACD

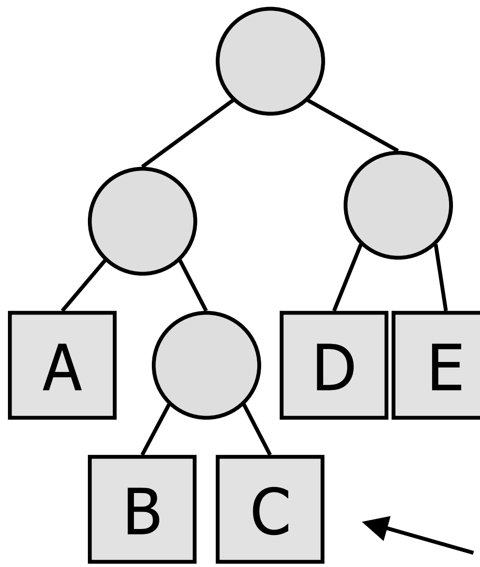
- Encoding:CD
- Decoding:A



Encoding and Decoding: ACD

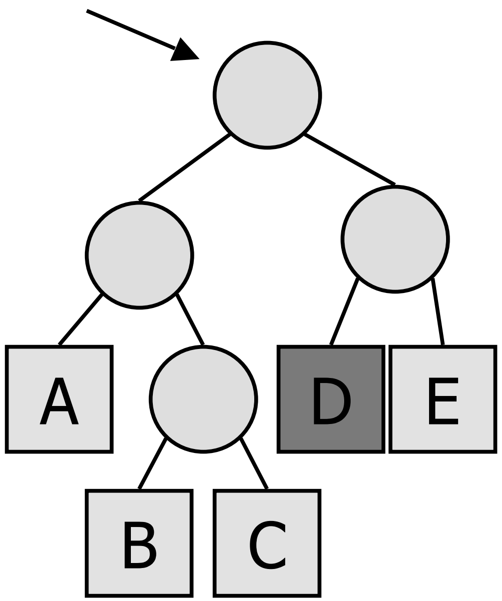
- Encoding:D
- Decoding:AC

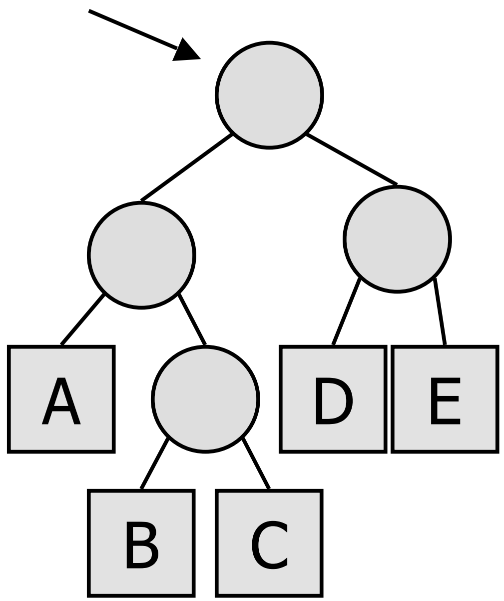




Encoding and Decoding: ACD

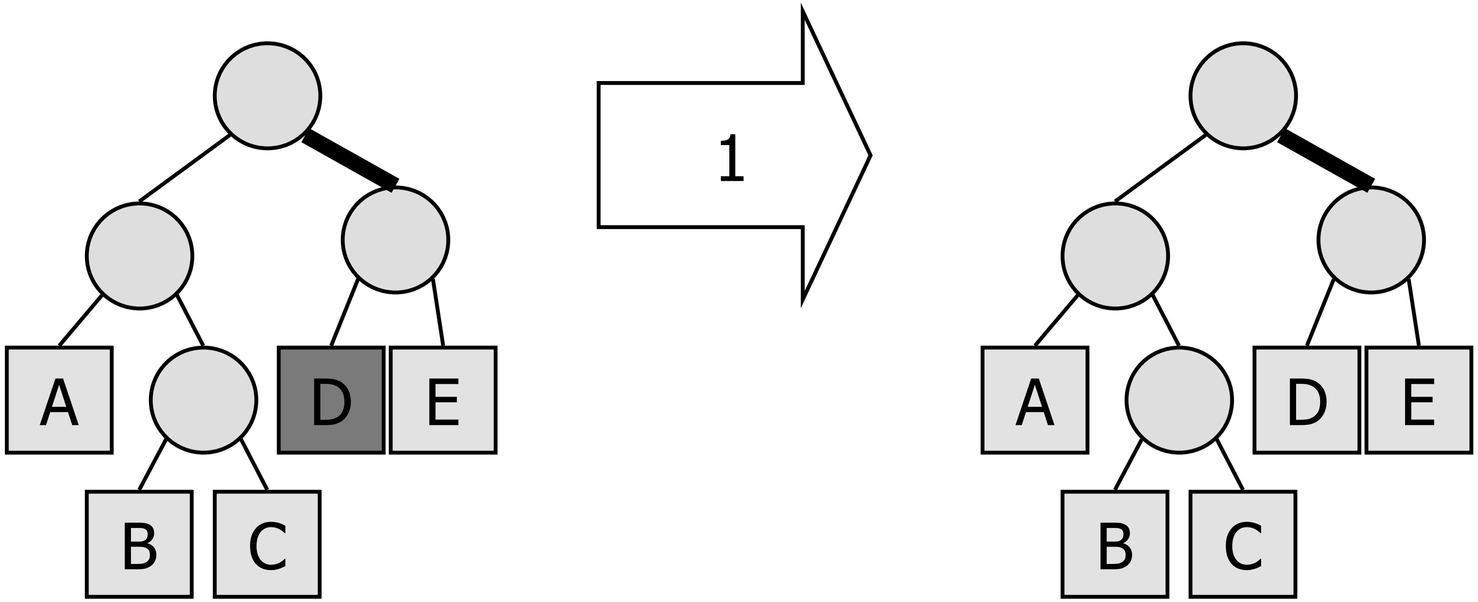
- Encoding:D
- Decoding:AC





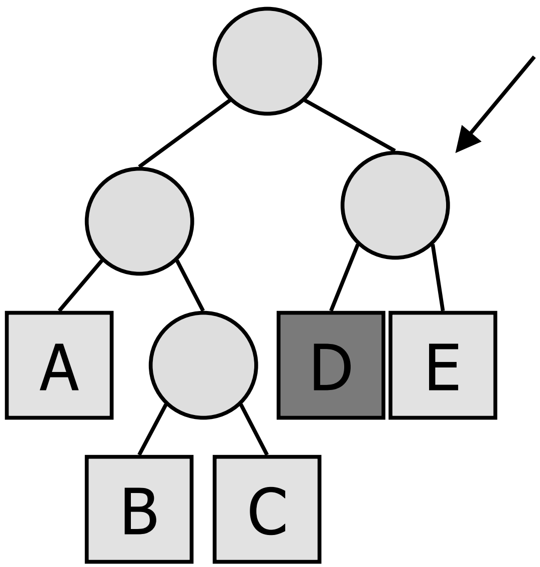
Encoding and Decoding: ACD

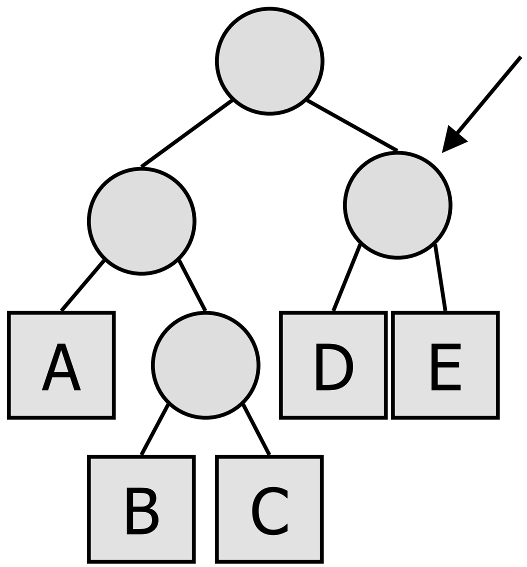
- Encoding:D
- Decoding:AC



Encoding and Decoding: ACD

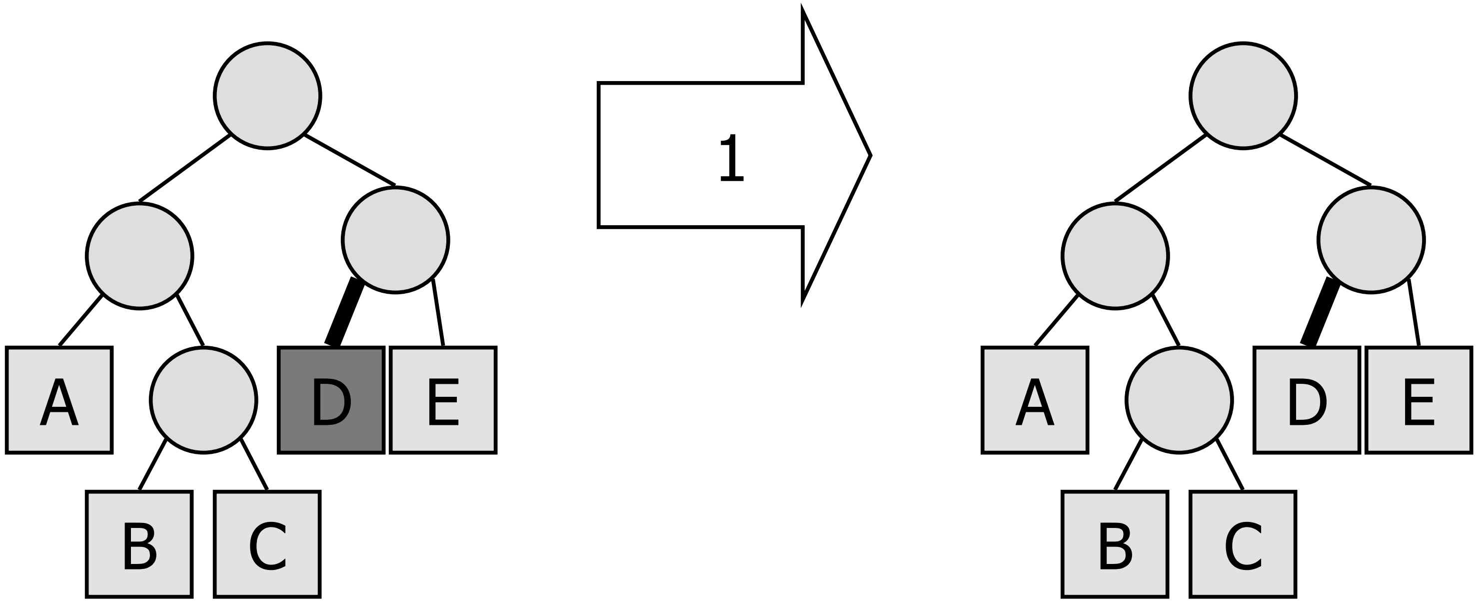
- Encoding:D
- Decoding:AC





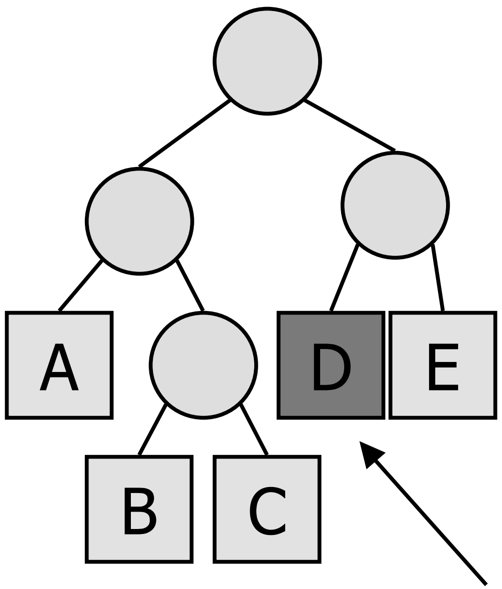
Encoding and Decoding: ACD

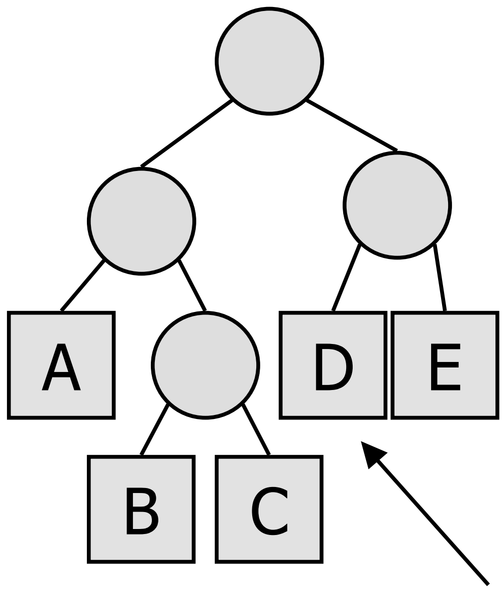
- Encoding:D
- Decoding:AC



Encoding and Decoding: ACD

- Encoding:
- Decoding:ACD





Back to Method I: Balanced Tree

- Method I was to used fixed length code words
- Each path from the root to a leaf is the same length: a balanced tree
- Balanced trees are good for worst case path length. Are they good for coding?
  - Yes, if you assume the worst case
  - But we can normally do better…

Statically optimal codes

- Want common symbols to have short codes
- This will make uncommon symbols have longer codes
  - In a tree with a fixed number of leave/symbols, moving one leaf/symbol closer to the root will move others further away

Huffman codes

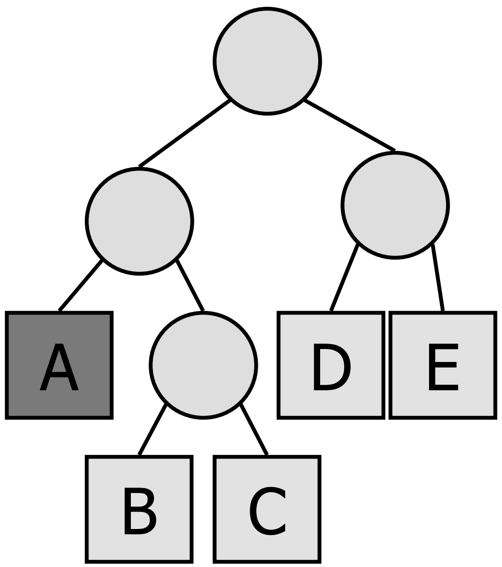
- From Shannon's information theory, The optimal static code assigns $-\log_2(p)$ bits to a symbol that occurs with probability p
- It is possible to make a Huffman code tree with this property
  - Will look at this later in the course

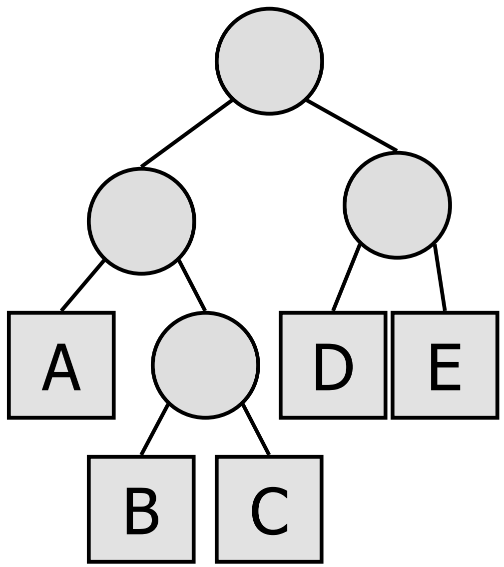
Adaptive Codes

- As long as the same change is made in both sending and receiving trees/codes, there is no reason why the tree/code must remain static
  - Send a character using the initial tree
  - Update the tree using that character
    - Can also be updated in the receiver as it already has the character
  - Send the next character

Encoding and Decoding: ACD

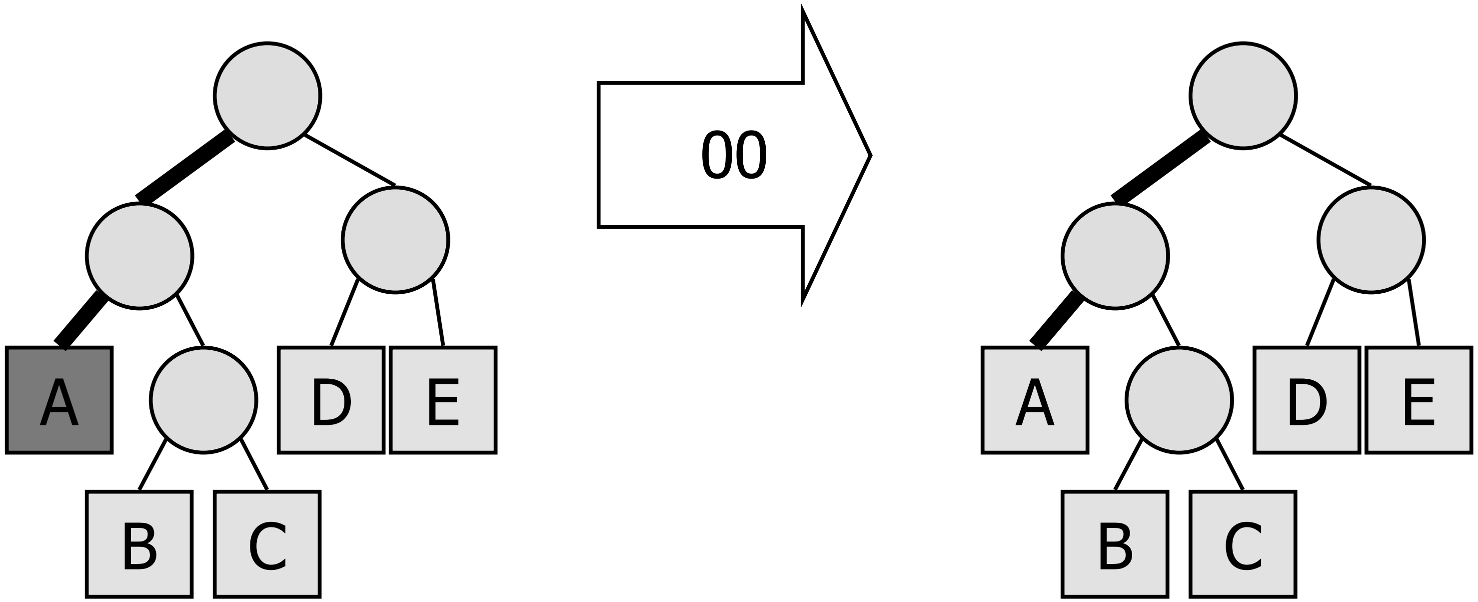
- Encoding:ACD
- Decoding:





Encoding and Decoding: ACD

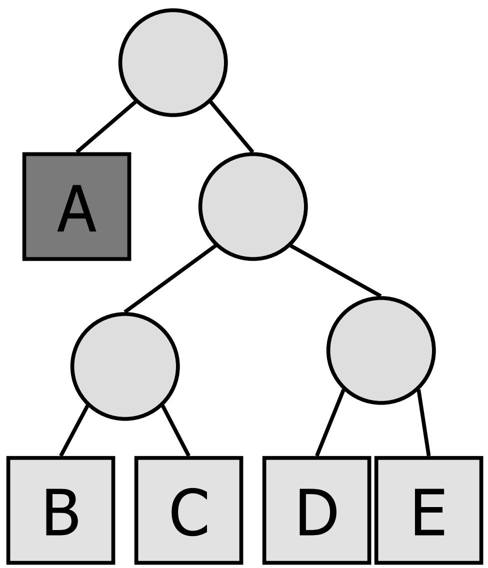
- Encoding:CD
- Decoding:A

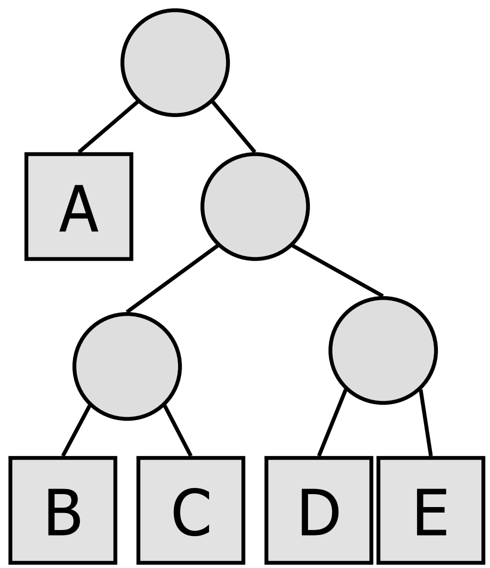


Encoding and Decoding: ACD

- Encoding:CD
- Decoding:A

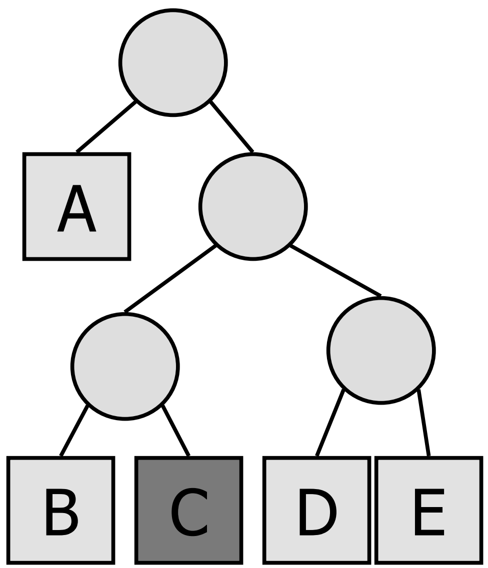
Make same change in both trees:
Rotate A's parent

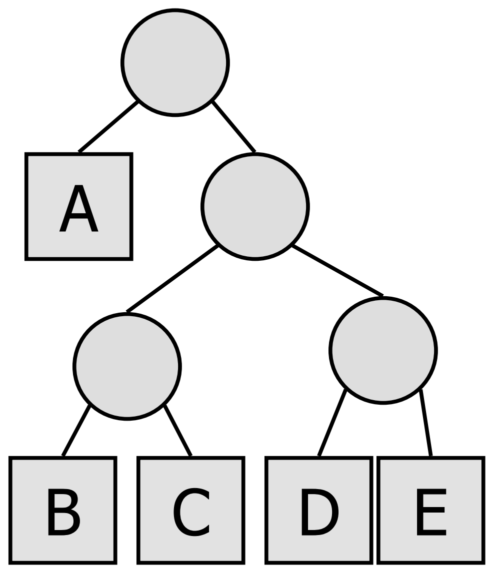




Encoding and Decoding: ACD

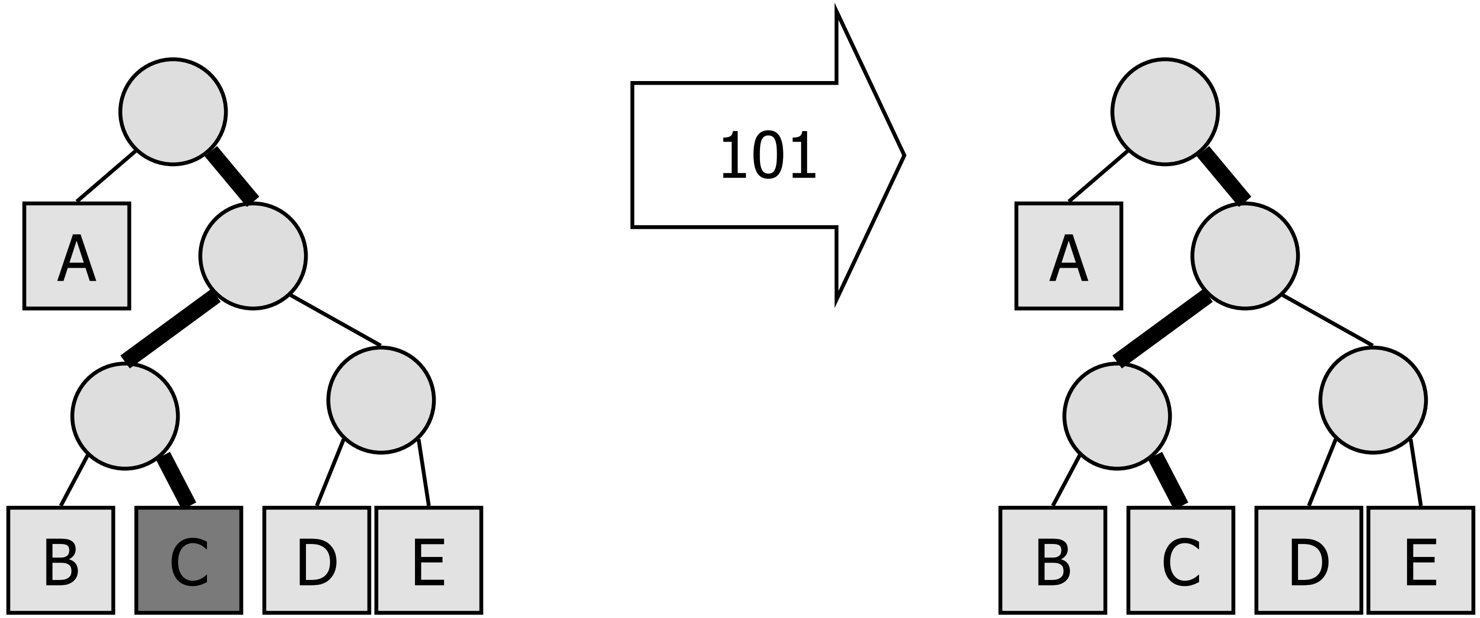
- Encoding:CD
- Decoding:A





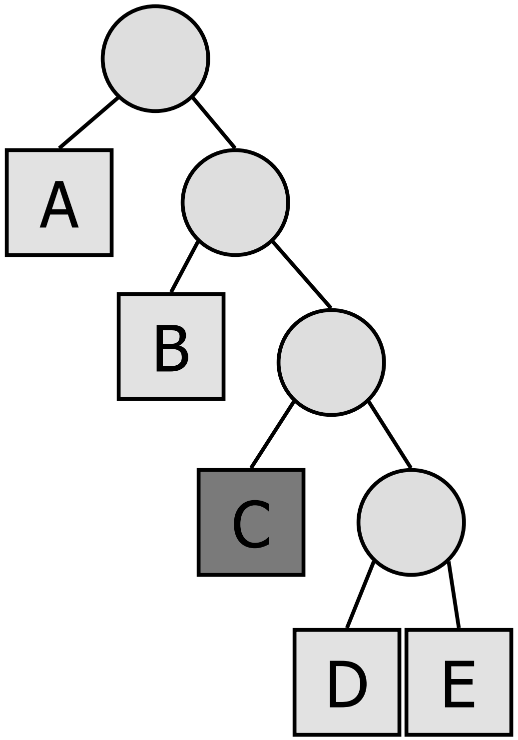
Encoding and Decoding: ACD

- Encoding:D
- Decoding:CA

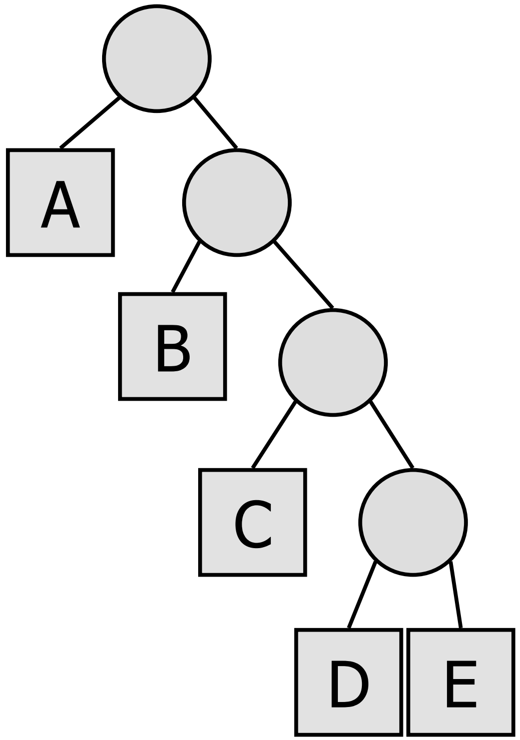


Encoding and Decoding: ACD

- Encoding:D

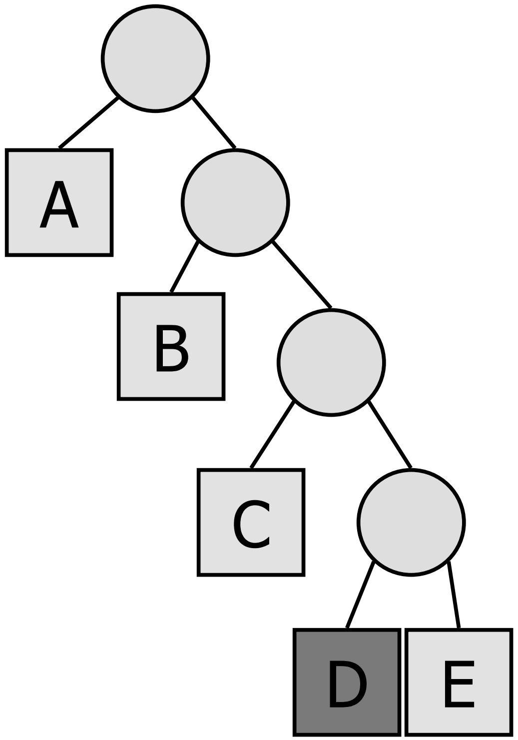


- Decoding:CA

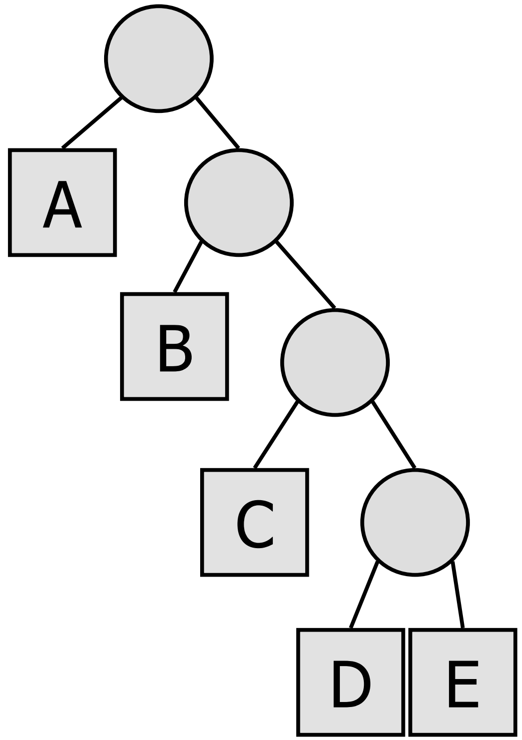


Encoding and Decoding: ACD

- Encoding:D



- Decoding:CA



Encoding and Decoding: ACD

- Encoding:
- Decoding:CAD