INTERNATIONAL JOURNAL OF COMMUNICATION SYSTEMS
*Int. J. Commun. Syst.* 2007; **20**:507–514

## SHORT COMMUNICATION

# Bit error rate performance of Haar wavelet based scale-code division multiple access (HW/S-CDMA) over the asynchronous AWGN channel

Oğuz Kucur[1,*,†], Ertan Öztürk[2] and Guillermo E. Atkin[3]

[1] *Department of Electronics Engineering, Gebze Institute of Technology, 41400 Gebze-Kocaeli, Turkey*
[2] *Department of Electrical and Electronics Engineering, Zonguldak Karaelmas University, 67100 Zonguldak, Turkey*
[3] *Electrical and Computer Engineering Department, Illinois Institute of Technology, Chicago, IL 60616, U.S.A.*

## SUMMARY

In this paper, we study a recently proposed multirate system, called wavelet based scale-code division multiple access (W/S-CDMA). W/S-CDMA depends on the code, time and scale orthogonality introduced by pseudo-noise (PN) sequences, and wavelets. In this system, the channel is partitioned into different scales, and each scale into time slots. In addition, the PN sequences are used in each scale to identify multiple users. In W/S-CDMA, each user is assigned a specific scale and PN sequence, and transmits its successive information symbols with its PN sequence and the wavelets in that scale. More symbols are transmitted in finer scales. We analyse the bit error rate performance of Haar wavelet based S-CDMA (HW/S-CDMA) over an asynchronous additive white Gaussian noise (AWGN) channel by using a conventional detector for deterministic PN sequences. The performance of the system is compared to that of an equivalent multirate CDMA (MR-CDMA) system for Gold and Kasami PN sequences. Results show that HW/S-CDMA outperforms MR-CDMA. In addition, because of its suitable format HW/S-CDMA is also capable of employing the optimal PN sequence families with limited number of sequences such as Kasami, Bent, etc. repeatedly in different scales. Copyright © 2006 John Wiley & Sons, Ltd.

Received 1 September 2005; Revised 1 April 2006; Accepted 1 May 2006

KEY WORDS: wavelets; multirate/multimedia communication; code division multiple access

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*Correspondence to: Oğuz Kucur, Department of Electronics Engineering, Gebze Institute of Technology, 41400 Gebze-Kocaeli, Turkey.
† E-mail: okucur@gyte.edu.tr

## 1. INTRODUCTION

Code division multiple access (CDMA) has been accepted as standard multiple access technique for the future generation wireless communication systems because of its attractive features such as its capacity, high spectrum efficiency, multipath fading resistance and flexibility to be modified for multirate traffic. Based on CDMA, there are mainly three schemes suggested for multirate communication: multi-code CDMA, multi-chip rate CDMA and multi-processing gain CDMA [1–4]. In multi-processing gain CDMA, different data rate users employ PN sequences of different lengths (different processing gains). However, the chip rate is fixed, i.e. all users spread their data over the same bandwidth. In this paper, we also study a kind of multi-processing gain CDMA scheme using Haar wavelet.

Wavelets have been used in many communication applications [5]. Among these applications, orthogonal variable spreading factor (OVSF) codes [6], which are variable length Hadamard (a special wavelet coefficient matrix) codes, enable a multi-processing gain scheme; however, in order to use the codes efficiently, dynamic assignment algorithms which increase complexity are employed. Wang and Cheng [7] have presented a multicarrier single rate CDMA system based on random PN sequences, and chip waveforms selected from wavelet packets. Öztürk *et al*. have designed a single rate quasi-synchronous CDMA system using random PN sequences and wavelets as chip waveforms, and obtained an optimum wavelet to improve the performance [8]. Kucur and Atkin have obtained a capacity improvement in scale time code division multiple access (STCDMA) [9], where different users communicate in each time slot of any scale. A multirate version of STCDMA, called wavelet based scale-CDMA (W/S-CDMA) has been proposed in Reference [10]. In W/S-CDMA, each transmitter uses a specific scale and is assigned a distinct PN sequence of different length that fits the time slots in its scale. Therefore, each user encodes its successive information symbols with time-shifted replicas of the same basic wavelet in its scale and spreads its scaled and translated wavelets (information symbols) with its PN sequence. In Reference [10], the performance of Haar W/S-CDMA (HW/S-CDMA) has been examined over a synchronous additive white Gaussian noise (AWGN) channel by using a decorrelating multi-user detector for any number of scales. In this paper, we analyse the bit error rate (BER) performance of 2-scale HW/S-CDMA over an asynchronous AWGN channel by using a conventional detector (i.e. a matched filter) for deterministic PN sequences. However, the analysis can be extended to higher scale formats. In the analysis, Gaussian approximation is used since it has been shown to be accurate for low signal-to-noise ratios (SNRs) and many users or for few users with a high processing gain [3].

Compared to the OVSF based multirate scheme, HW/S-CDMA can employ many users easily by assigning the same PN codes repeatedly in different scales. In addition, practical PN sequences such as Gold and Kasami PN sequence sets used in this work have better randomness properties than Hadamard based OVSF codes. Although random codes are ideal in terms of autocorrelation and crosscorrelation properties, they are not practical since they also need to be transmitted together with the information symbols [11]. Therefore, to be practical we use deterministic PN sequences such as Gold and Kasami while many multirate CDMA systems [1–4] and wavelet based single-rate systems [7, 8] generally work on random codes.

The paper is organized as follows. The performance of the system is analysed in Section 2. Numerical results are given in Section 3. Section 4 summarizes conclusions.

DOI: 10.1002/dac

## 2. PERFORMANCE ANALYSIS

In general, the signal for the $i$th user in the $m$th scale of HW/S-CDMA is expressed as

$$s_{m,i}(t) = \sum_{n=-\infty}^{\infty} b_{m,i}^{n} w_{m,n}(t) c_{m,n,i}(t) = \sqrt{E} \sum_{n=-\infty}^{\infty} b_{m,i}^{n} w_{m,n}(t) p_{il}(t - nT/2^{m}) \cos\{\omega_{\mathrm{c}}(t - nT/2^{m}) + \varphi_{m,i}\} \tag{1}$$

where $b_{m,i}^{n}$ is the information bit transmitted in the $n$th slot of the $m$th scale by the $i$th user, $w_{m,n}(t)$ is the Haar wavelet in that scale and slot, and $c_{m,n,i}(t)$ is the carrier signal. $E$ is the signal amplitude, $p_{il}(t)$ is the $i$th signature waveform of $l = 2^{1-m}$ periods, $\omega_{\mathrm{c}}$ is the carrier frequency and $\varphi_{m,i}$ is the carrier phase for the $i$th user in the $m$th scale. By using the Haar wavelet embedded signature waveform $\hat{p}_{il}(t)$ [5], the total 2-scale HW/S-CDMA signal for asynchronous communication is given as

$$\begin{aligned} s(t) &= \sum_{m=0}^{1} \sum_{i=1}^{L} s_{m,i}(t - \tau_{m,i}) \\ &= \sum_{m=0}^{1} \sum_{n} \sum_{i=1}^{L} \sqrt{2^{m} E} b_{m,i}^{n} \hat{p}_{il}(t - nT/2^{m} - \tau_{m,i}) \cos\{\omega_{\mathrm{c}}(t - nT/2^{m}) + \theta_{m,i}\} \end{aligned} \tag{2}$$

where $\tau_{m,i}$ is the time delay of the $i$th user in the $m$th scale, $L$ is the number of users (PN sequences) in each scale and $\theta_{m,i} = \varphi_{m,i} - \omega_{\mathrm{c}} \tau_{m,i}$. We assume that the phase shifts $\theta_{m,i}$ are uniformly distributed over $[0, 2\pi]$, and the time delays $\tau_{m,i}$ are uniformly distributed over [0, T/2] since the width of the time slots in the second scale is T/2. The total asynchronous HW/S-CDMA signal in (2) is corrupted by AWGN. Then, the received signal can be expressed by $r(t) = s(t) + n(t)$, where $n(t)$ is the AWGN with power spectral density $N_0/2$.

We first decode the information bit of the first user in the first scale, i.e. $m = 0$, $n = 0$, $i = 1$, $l = 2$ by assuming that $\tau_{0,1} = 0$ and $\theta_{0,1} = 0$ without loss of generality. We also assume that each user has the same received energy, $ET/2$ without loss of generality, i.e. equally received energy policy is applied whereas user powers differ on the basis of rates as in other multi-processing gain schemes [2]. The information bits $\{\mp 1\}$ are equally probable. In addition, the time delays $\tau_{m,i}$, the phase shifts $\theta_{m,i}$, the information bits $b_{m,i}^{n}$ and the noise components are independent from each other for any $m$, $i$ and $n$. Then, by assuming that the information bit of the desired user, $b_{0,1}^{0}$ is $+1$ without loss of generality, the matched filter output will be

$$\begin{aligned} r_{0,1}^{0} = \int_{0}^{T} r(t) \hat{p}_{12}(t) \cos(\omega_{\mathrm{c}} t)\, \mathrm{d}t = \frac{\sqrt{E}T}{2} &+ \underbrace{\frac{\sqrt{E}}{2} \sum_{i=2}^{L} [b_{0,i}^{-1} R_{i1}^{(1)}(\tau_{0,i}) + b_{0,i}^{0} R_{i1}^{(2)}(\tau_{0,i})] \cos(\theta_{0,i})}_{I_d} \\ &+ \underbrace{\sqrt{E/2} \sum_{i=1}^{L} [\{b_{1,i}^{-1} - b_{1,i}^{0}\} R_{i1}^{(3)}(\tau_{1,i}) + \{b_{1,i}^{1} - b_{1,i}^{0}\} \hat{R}_{i1}^{(3)}(\tau_{1,i})] \cos(\theta_{1,i})}_{I_f} \\ &+ \underbrace{\int_{0}^{T} n(t) \hat{p}_{12}(t) \cos(\omega_{\mathrm{c}} t)\, \mathrm{d}t}_{n_{0,1}^{0}} \end{aligned} \tag{3}$$

DOI: 10.1002/dac

where the first term is the desired user's component, the second term is the multiple access interference (MAI) from the users sharing the desired user's scale (and slot), and the third term is the MAI from the finer (second) scale users. The fourth term in (3), $n^0_{0,1}$ is the noise component, which is of zero mean and has a variance of $N_0T/4$. $R^{(k)}_{ij}(\tau)$ are partial crosscorrelation functions.

The probability of bit error for the first scale users is obtained as [5]

$$P_{0,1,(0)} = \frac{1}{2} erfc\left\{ \left( \frac{1}{12N^3} \sum_{i=2}^{L} Z^{(1)}_{i1} + \frac{1}{3N^3} \sum_{i=1}^{L} \left( Z^{(2)}_{i1} - Z^{(3)}_{i1} \right) + \frac{1}{\gamma_b} \right)^{-1/2} \right\} \tag{4}$$

where $\gamma_b = (ET/2)/N_0$ is the SNR per bit, and

$$\begin{aligned} Z^{(1)}_{i1} &= 1.5[C^{(1)}_{i1}(0)]^2 + 2\sum_{k=1}^{N-1} \left\{ [C^{(1)}_{i1}(k)]^2 + [C^{(1)}_{i1}(k-2N)]^2 \right\} + \sum_{k=0}^{N-1} C^{(1)}_{i1}(k) C^{(1)}_{i1}(k+1) \\ &\quad + \sum_{k=1}^{N-1} C^{(1)}_{i1}(k-2N) C^{(1)}_{i1}(k+1-2N) \\ Z^{(2)}_{i1} &= 2\sum_{k=1-N}^{N-1} [C^{(2)}_{i1}(k)]^2 + \sum_{k=1-N}^{N-1} C^{(2)}_{i1}(k) C^{(2)}_{i1}(k+1) \\ Z^{(3)}_{i1} &= 2\sum_{k=1}^{N-1} C^{(2)}_{i1}(k) C^{(2)}_{i1}(k-N) + \frac{1}{2}\sum_{k=1}^{N-2} C^{(2)}_{i1}(k+1) C^{(2)}_{i1}(k-N) \\ &\quad + \frac{1}{2}\sum_{k=0}^{N-1} C^{(2)}_{i1}(k) C^{(2)}_{i1}(k+1-N) \end{aligned} \tag{5}$$

$C^{(1)}_{i1}(k) = C_{i1}(k, 2N, \hat{\mathbf{h}}_\mathbf{i}, \hat{\mathbf{h}}_\mathbf{1})$ and $C^{(2)}_{i1}(k) = C_{i1}(k, N, \hat{\mathbf{h}}_\mathbf{i}, \mathbf{h}_\mathbf{1})$ in (5), where $C_{ij}(k, lN, \mathbf{h}_\mathbf{i}, \mathbf{h}_\mathbf{j})$ is the discrete aperiodic crosscorrelation function of two PN sequences $\mathbf{h}_\mathbf{i}$ and $\mathbf{h}_\mathbf{j}$ over $l$ periods ($N$ is the period) and $\hat{\mathbf{h}}_\mathbf{i}$ is the Haar wavelet embedded sequence [5].

Similarly, the probability of bit errors for the first and second slots of the second scale can be obtained as [5].

$$P_{1,1,(n)} = \frac{1}{2} erfc\left\{ \left( \frac{1}{3N^3} \sum_{i=2}^{L} Z^{(4)}_{i1} + \frac{1}{6N^3} \sum_{i=1}^{L} (Z^{(5)}_{i1} - nZ^{(6)}_{i1}) + \frac{1}{\gamma_b} \right)^{-1/2} \right\} \tag{6}$$

where $n = 0$ and $n = 1$ for the first and second slots, respectively, and

$$\begin{aligned} Z^{(4)}_{i1} &= 2\sum_{k=1-N}^{N-1} [C^{(2)}_{i1}(k)]^2 + \sum_{k=1-N}^{N-1} C^{(2)}_{i1}(k) C^{(2)}_{i1}(k+1) \\ Z^{(5)}_{i1} &= 2\sum_{k=1-N}^{N-1} [C^{(3)}_{i1}(k)]^2 + \sum_{k=1-N}^{N-1} C^{(3)}_{i1}(k) C^{(3)}_{i1}(k+1) \\ Z^{(6)}_{i1} &= 4\sum_{k=1}^{N-1} C^{(3)}_{i1}(k) C^{(3)}_{i1}(k-N) + \sum_{k=1}^{N-2} C^{(3)}_{i1}(k+1) C^{(3)}_{i1}(k-N) + \sum_{k=0}^{N-1} C^{(3)}_{i1}(k) C^{(3)}_{i1}(k+1-N) \end{aligned} \tag{7}$$

In (7), $C^{(3)}_{i1} = C_{i1}(k, N, \mathbf{h}_\mathbf{i}, \hat{\mathbf{h}}_\mathbf{1})$ [5].

## 3. NUMERICAL RESULTS

In this section, we will compare the BER performance of HW/S-CDMA to that of an equivalent multirate CDMA (MR-CDMA) for Gold and Kasami PN sequences. MR-CDMA is the equivalent system without wavelets. In other words, in dual rate MR-CDMA, high rate users use one period of the sequences and low rate users use two periods of the same sequences. The same assumptions and operating conditions as those of HW/S-CDMA are valid. Therefore, the analysis of HW/S-CDMA can be exploited to determine the BER performance of MR-CDMA. For the following figures, in the numerical calculations using (4) and (6), the first $L$ sequences from Gold and Kasami PN sequence sets are used and average performance is displayed.

Figure 1 shows the performance of the extended Gold PN sequences of period $N=64$ for $L=10$ users in each rate of both systems. As depicted in Figure 1 for a total of 20 users (30 symbols), the first scale users of HW/S-CDMA outperform the low rate users of MR-CDMA approximately 1.5 dB at a BER of $10^{-3}$. At the same BER, the SNR value of the second scale users of HW/S-CDMA is almost 0.5 dB lower than that of the high rate users of MR-CDMA. The SNR difference between two rates is about 3 dB at a BER of $10^{-3}$ for both systems.

Figure 2 depicts the performance of the extended Gold PN sequences of period $N=128$ for $L=20$ users in each rate of both systems. As seen in Figure 2, for a total of 40 users (60 symbols), the performance of the low rate users of MR-CDMA degrades approximately 1 dB at a BER of $10^{-3}$ as compared to that of the first scale users of HW/S-CDMA. At the same BER, the SNR value of the second scale users of HW/S-CDMA is almost 0.2 dB lower than that of the

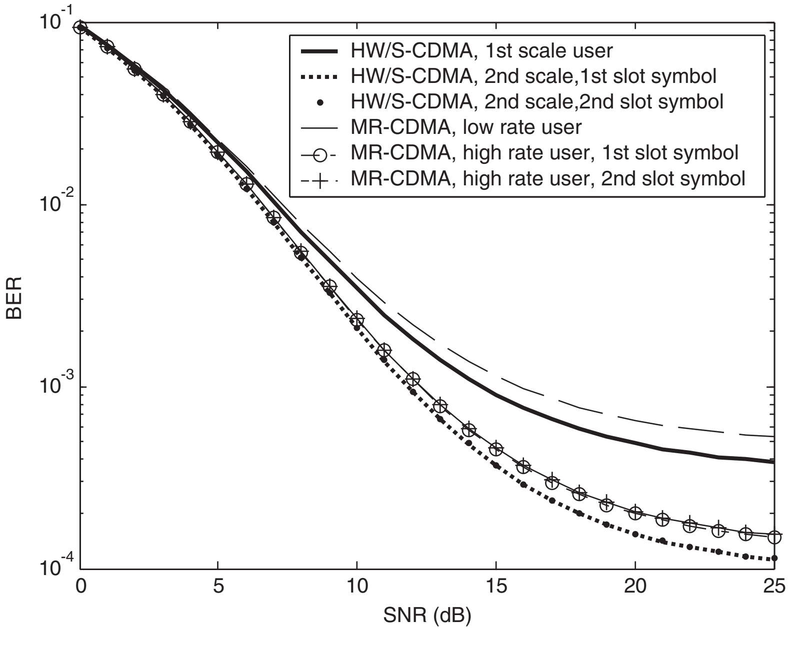

Figure 1. Performance comparison between HW/S-CDMA and MR-CDMA for $L=10$ Gold PN sequences of period 64.

DOI: 10.1002/dac

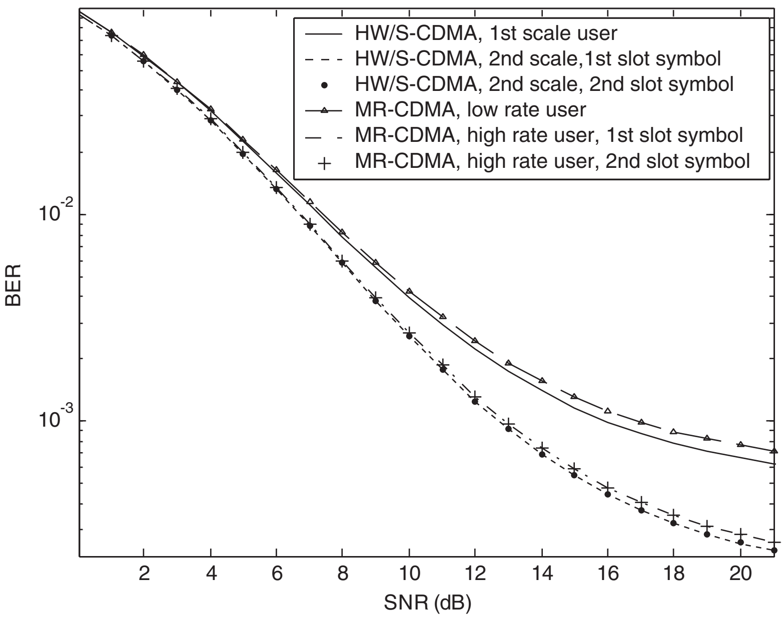

Figure 2. Performance comparison between HW/S-CDMA and MR-CDMA for $L = 20$ Gold PN sequences of period 128.

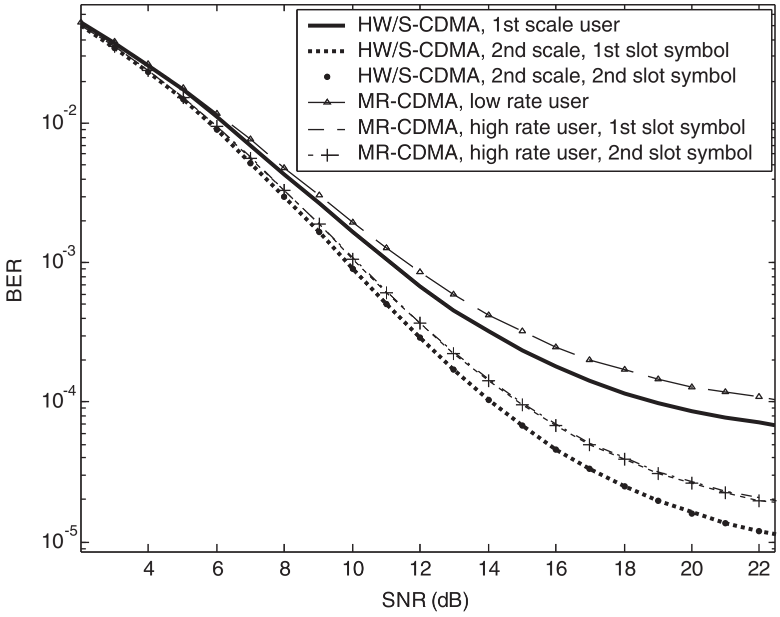

Figure 3. Performance comparison between HW/S-CDMA and MR-CDMA for $L = 8$ Kasami PN sequences of period 64.

high rate users of MR-CDMA. The SNR differences between two rates are 3 and 4 dB for HW/S-CDMA and MR-CDMA, respectively, at a BER of $10^{-3}$.

Because of the reuse capability provided by the scales, both systems are especially suitable for optimal sequence families such as Bent and Kasami, which have limited number of sequences. Kasami PN sequences cannot be used in other multi-processing gain schemes since they do not exist for all lengths. For this reason, the performance of both systems for the extended Kasami PN sequences of period $N = 64$ is depicted in Figure 3. As observed in the figure, for a total of 16 users (24 symbols), the SNR performance of the first scale users of HW/S-CDMA is 4 dB better than that of the low rate users of MR-CDMA at a BER of $10^{-4}$. At the same BER, the performance of the second scale users of HW/S-CDMA is approximately 0.8 dB better than that of the high rate users of MR-CDMA. The SNR differences between two rates are 5 and 8 dB for HW/S-CDMA and MR-CDMA, respectively, at a BER of $10^{-4}$.

As observed in Figures 1–3, the low rate users of HW/S-CDMA always outperform considerably those of MR-CDMA for all processing gains. The performance of the high rate users of HW/S-CDMA is better than that of MR-CDMA for low processing gains. In addition, there is a performance difference between two rates for both systems.

## 4. CONCLUSIONS

In this paper, we have analysed the performance of 2-scale HW/S-CDMA over the asynchronous AWGN channel for deterministic PN sequences. We compared the performance of HW/S-CDMA to that of MR-CDMA for Gold and Kasami PN sequences. Results show that HW/S-CDMA outperforms MR-CDMA for the same number of users (and symbols) and bandwidth occupancy. For high rate users, HW/S-CDMA is better than MR-CDMA for low processing gains. For low rate users, HW/S-CDMA outperforms MR-CDMA for all processing gains. In addition, HW/S-CDMA is especially useful to employ optimal Kasami PN sequences because of its format which enables the assignment of the same PN sequences with multiple periods repeatedly in different scales. Future work should consider the analysis of HW/S-CDMA over fading channels.

## REFERENCES

1. Roy S, Yan H. Blind channel estimation in multi-rate CDMA systems. *IEEE Transactions on Communication* 2002; **50**(6):995–1004.
2. Yao S, Geraniotis E. Multirate CDMA for multi-media wireless communication. *SPIE* 1995; **2601**:60–71.
3. Ottosson T, Svensson A. On schemes for multirate support in DS-CDMA systems. *Journal of Wireless Personal Communication* 1998; **6**(3):265–287.
4. Chen J, Mitra U. Optimum near-far resistance for dual-rate DS/CDMA signals: random signature sequence analysis. *IEEE Transaction on Information Theory* 1999; **45**(7):2434–2447.
5. Kucur O, Öztürk E, Atkin GE. Performance analysis of Haar wavelet based scale-code division multiple access over the asynchronous AWGN channel. *Technical report GYTE-ELM-2006-001*. http://www.gyte.edu.tr/gytenet/Dosya/102/teknikraporlar/index.htm
6. Cheng R-G, Lin P. OVSF code channel assignment for IMT-2000. *IEEE VTC*, vol. 3, 15–18 May 2000; 2188–2192. DOI: 10.1109/VETECS.2000.851660
7. Wang R, Cheng SX. Performance of MC-CDMA based on wavelet packets in Rayleigh multipath fading channel. *IEE Electronics Letters* 2000; **36**(12):1070–1072.
8. Öztürk E, Kucur O, Atkin GE. Performance of optimum wavelet waveform for DS-CDMA chip waveform over QS-AWGN channel. *International Journal of Communication Systems* 2006; **19**(1):1–16.

DOI: 10.1002/dac

9. Kucur O, Atkin GE. Performance of scale-time-code division multiple access over the synchronous AWGN channel. *International Journal of Communication Systems* 2000; **13**(6):505–516.
10. Kucur O, Atkin GE. Performance of Haar wavelet based scale-code division multiple access (HW/S-CDMA) using decorrelating multi-user detector. *Journal of Computers and Electrical Engineering* 2005; **31**(7):468–484.
11. Haykin S, Moher M. *Modern Wireless Communications*. Prentice-Hall: Englewood Cliffs, NJ, 2005.

## AUTHORS' BIOGRAPHIES

**Oğuz Kucur** got his BS degree in Electronics and Telecommunication Engineering from Istanbul Technical University, Istanbul, Turkey in 1988. He received his MS and PhD degrees both in Electrical and Computer Engineering from Illinois Institute of Technology (IIT), Chicago, U.S.A., in 1992 and 1998, respectively. From 1996 to 1998 he was a teaching assistant at IIT. He was an assistant professor in the Department of Electrical Engineering at South Dakota State University in the 1998–1999 academical year. He has been working as an assistant professor in the Department of Electronics Engineering at Gebze Institute of Technology, Turkey since October 1999. His research interests are wireless communication systems, spread spectrum systems, multiple access techniques, and code-division multiple access systems.

**Ertan Öztürk** received his BS degree in Electrical and Electronics Engineering from Gazi University, Ankara, Turkey in 1992, and his MS and PhD degrees both in Electrical and Computer Engineering from Illinois Institute of Technology, Chicago, in 1995 and 2001, respectively. He worked as a senior system engineer from 2000 to 2002 at Motorola, Illinois, where he worked on UMTS system simulations and planning.

He has been working as an assistant professor in the Department of Electrical and Electronics Engineering at Zonguldak Karaelmas University, Turkey since July 2002. His research interests are in the area of wireless communication systems including code division multiple access systems and wireless local area networks.

**Guillermo E. Atkin** joined Illinois Institute of Technology (IIT) in 1986. From 1974 to 1981 he was a full-time lecturer in the Electrical Engineering Department at the University Federico Santa Maria and a consultant for the telecommunications industry. In 1982, he was a PhD student in the Electrical Engineering Department at the University of Waterloo, Ontario, Canada. During his studies he was a research assistant for the Natural Sciences and Engineering Research Council of Canada.

His current interests are in the area of bandwidth efficient coding and modulation techniques for digital mobile systems, wireless communication systems, and optical communication systems. Dr Atkin is the Director of the Digital Communication Systems Laboratory at IIT. Dr Atkin has published more than 50 technical papers and reports in the communications area.

DOI: 10.1002/dac